Staggered Chiral Perturbation Theory with Heavy-Light Mesons

C. Aubin and C. Bernard

Washington University,

St. Louis, MO USA

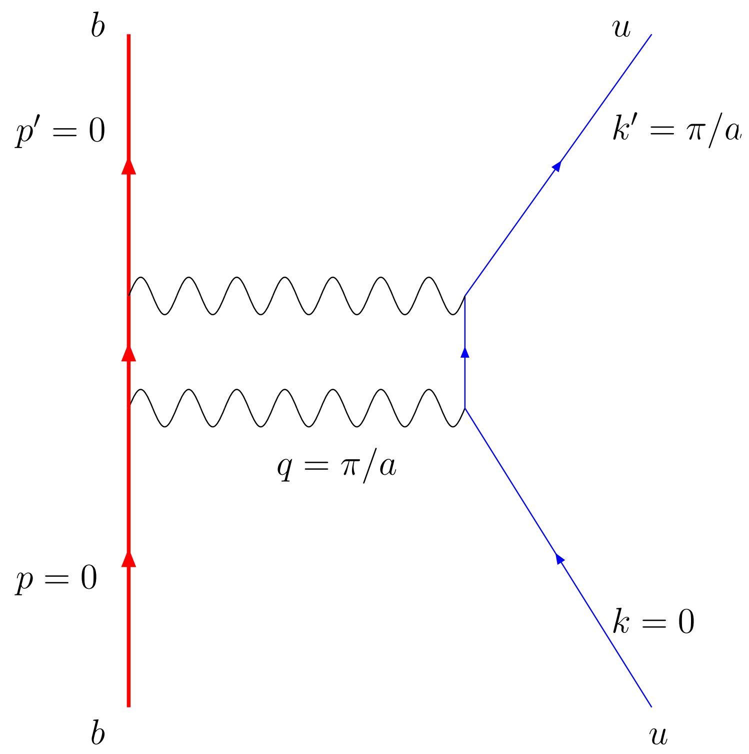
Outline

- Effective Theory of heavy-lights and pions
  - Heavy Quark Effective Theory (HQET)
  - Staggered Chiral Perturbation Theory (SχPT)
  - Taste breaking with heavy-lights
- f_B NLO Calculation
- Form factors for $B(D) \to \pi(K)\ell\nu$

Heavy-Lights on the Lattice

- One light staggered quark
- One heavy quark:
  - m_Q is large but $\neq \infty$; else taste violations can be quite large.
  - But $1/m_Q$ corrections are small.

$$a \approx .12\text{fm} \Rightarrow \pi/a \approx 5.2\text{GeV}$$
b quark: $5\text{GeV} \longrightarrow 7.2\text{GeV}$



- 16 tastes of pions (in $SO(4)$ representations: P, A, T, V, S)
- 4 tastes of heavy-lights.

Heavy-Lights on the Lattice

- We are parametrizing only the taste violations in heavy-light quantities.
- Discretization errors that would arise in the heavy sector are not considered.
- If we were to have a highly improved heavy quark, these errors could be small and unimportant.
- In practice, the discretization errors would have to be extrapolated away.

HQET & SχPT

- The heavy-lights are combined into a single field H:

$$H_a = \frac{1+\not{v}}{2}\left[\gamma^\mu B^*_{\mu a} - \gamma_5 B_a\right] \qquad \overline{H}_a \equiv \gamma_0 H_a^\dagger \gamma_0$$

- Light mesons: $\Sigma = \sigma^2 = \exp(i\Phi/f)$, with (for 3 flavors of light quarks)

$$\Phi = \begin{pmatrix} U & \pi^+ & K^+ \\ \pi^- & D & K^0 \\ K^- & \bar{K}^0 & S \end{pmatrix}, \quad U = U_a T_a\,, \quad K^+ = K_a^+ T_a\,, \quad \textit{etc.},$$

- Under chiral $SU(12)_L \times SU(12)_R$:

$$H_a \to H_b U^\dagger_{ba} \qquad \overline{H}_a \to U_{ab}\overline{H}_b$$
$$\sigma \to L\sigma U^\dagger = U\sigma R^\dagger \qquad \Sigma \to L\Sigma R^\dagger$$

HQET & SχPT

- $\mathcal{L}$ is an expansion in
  - $m_\pi \sim \sqrt{m_q}$; m_q is a light quark mass
  - a^2, the lattice spacing
  - k ($p_B = m_Q v + k$), the heavy-light residual momentum

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4$$

$$\begin{aligned}
\mathcal{L}_2 &= i\text{tr}_D[\overline{H}_a v^\mu (\delta_{ab}\partial_\mu + i\mathbb{V}^{ba}_\mu) H_b] + g_\pi \text{tr}_D(\overline{H}_a H_b \gamma^\nu \gamma_5 \mathbb{A}^{ba}_\nu) + \mathcal{L}_{S\chi PT} \\
\mathcal{L}_4 &= g_1 \text{tr}_D(\overline{H}_a H_b)\mathcal{M}^+_{ba} + g_2 \text{tr}_D(\overline{H}_a H_a)\mathcal{M}^+_{bb} + \mathcal{L}_{LG} - a^2 \mathcal{V}_H
\end{aligned}$$

$$\mathbb{V}_\mu = \frac{i}{2}\left[\sigma^\dagger \partial_\mu \sigma + \sigma \partial_\mu \sigma^\dagger\right] \qquad \mathbb{A}_\mu = \frac{i}{2}\left[\sigma^\dagger \partial_\mu \sigma - \sigma \partial_\mu \sigma^\dagger\right]$$

$$\mathcal{M}^+ = \sigma \mathcal{M} \sigma + \sigma^\dagger \mathcal{M} \sigma^\dagger$$

Discrete Symmetry

- Staggered quarks have a discrete symmetry, given by

$$q \longrightarrow (1 \otimes \xi_\mu) q$$

- At the meson level, this takes the form

$$\Sigma \longrightarrow \xi_\mu^{(3)} \Sigma \xi_\mu^{(3)} \qquad \sigma \longrightarrow \xi_\mu^{(3)} \sigma \xi_\mu^{(3)}$$
$$H \longrightarrow H \xi_\mu^{(3)} \qquad \overline{H} \longrightarrow \xi_\mu^{(3)} \overline{H}$$

- $\mathcal{L}$ is invariant under this symmetry, which implies that matrix elements (A) bilinear in the heavy-light fields can be written as an average over tastes:

$$A_{ab} = \frac{1}{4} \sum_a A_{aa}$$

NLO calculation of f_B

- The B decay constant can be extracted from the matrix element

$$\frac{1}{4}\sum_a \left\langle 0 \left| L^{\mu}_{x,a} \right| B_{x,a}(v) \right\rangle = -i f_{B_x} m_{B_x} v^{\mu} \ ,$$

where $L^{\mu}_{x,a}$ is the axial current which destroys a $B_{x,a}$ meson. The corresponding chiral operator is

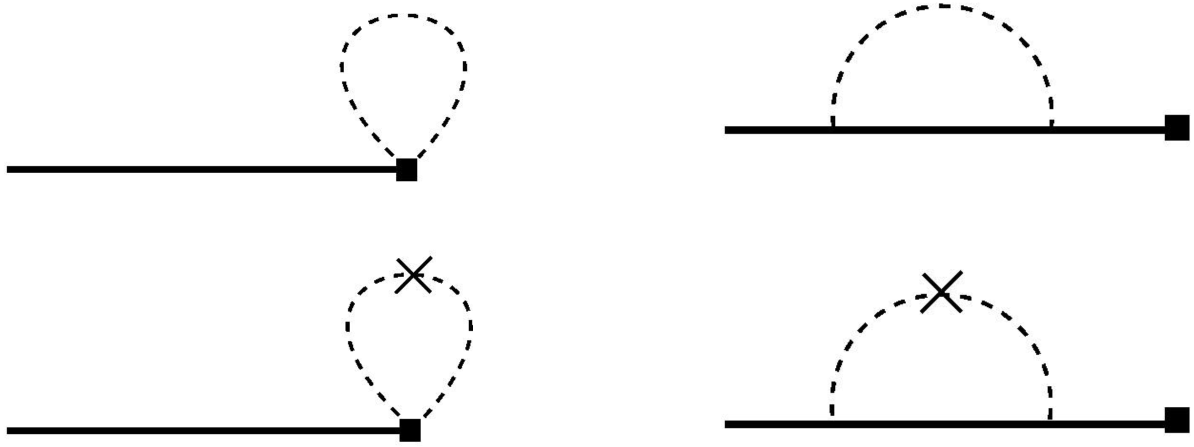
$$L^{\mu}_{x,a} = \frac{i\kappa}{2} \mathrm{tr}_D \left[\gamma^{\mu}(1-\gamma_5)(\mathcal{P}_x H_b)\right] \sigma_{ba}$$

- We will write the decay constant as:

$$f_{B_x} = \frac{\kappa}{\sqrt{m_{B_x}}} \left(1 + \frac{1}{16\pi^2 f^2} \delta f_{B_x} \right) \ ,$$

NLO calculation of f_B

- The only non-zero diagrams are:



- Crosses are one or more hairpin insertions from $\mathcal{L}_{S\chi PT}$:

$$\boxed{-ia^2\delta'_V\,(\text{taste-vector});\quad -ia^2\delta'_A\,(\text{taste-axial});\quad -i4m_0^2/3,\,(\text{taste-singlet})}$$

- Moving from $4 \to 1$ tastes per flavor is no different than in the calculations for light meson quantities.

Partially quenched f_{B_x}

$$\delta f_{B_x} = \frac{1+3g_\pi^2}{2}\Bigg\{-\frac{1}{16}\sum_{N,t}\ell(m_{N_t}^2) + \frac{1}{3}\sum_{j_I\in\mathcal{M}_I^{(2)}}\frac{\partial}{\partial m_{X_I}^2}\left[R_{j_I}^{[3,3]}(\mathcal{M}_I^{(1)};\mathcal{M}_I^{(2)})\ell(m_{j_I}^2)\right]$$

$$+a^2\delta_V'\sum_{j_V\in\mathcal{M}_V^{(3)}}\frac{\partial}{\partial m_{X_V}^2}\left[R_{j_V}^{[4,3]}(\mathcal{M}_V^{(1)};\mathcal{M}_V^{(3)})\ell(m_{j_V}^2)\right] + [V\to A]$$

$$+c_1(m_u+m_d+m_s) + c_2 m_x + c_a a^2\Bigg\}$$

$$\ell(m^2) = m^2\ln m^2 + \text{finite volume corrections}$$

$$\mathcal{M}^{(1)} = \{m_U^2, m_D^2, m_S^2\} \qquad \mathcal{M}_I^{(2)} = \{m_{\pi_I^0}^2, m_{\eta_I}^2\}$$

$$\mathcal{M}_V^{(3)} = \{m_{\pi_I^0}^2, m_{\eta_I}^2, m_{\eta_V'}^2\}$$

- The R_j are the residues of the poles of the disconnected flavor-neutral propagators.

f_B for $2{+}1$ flavors

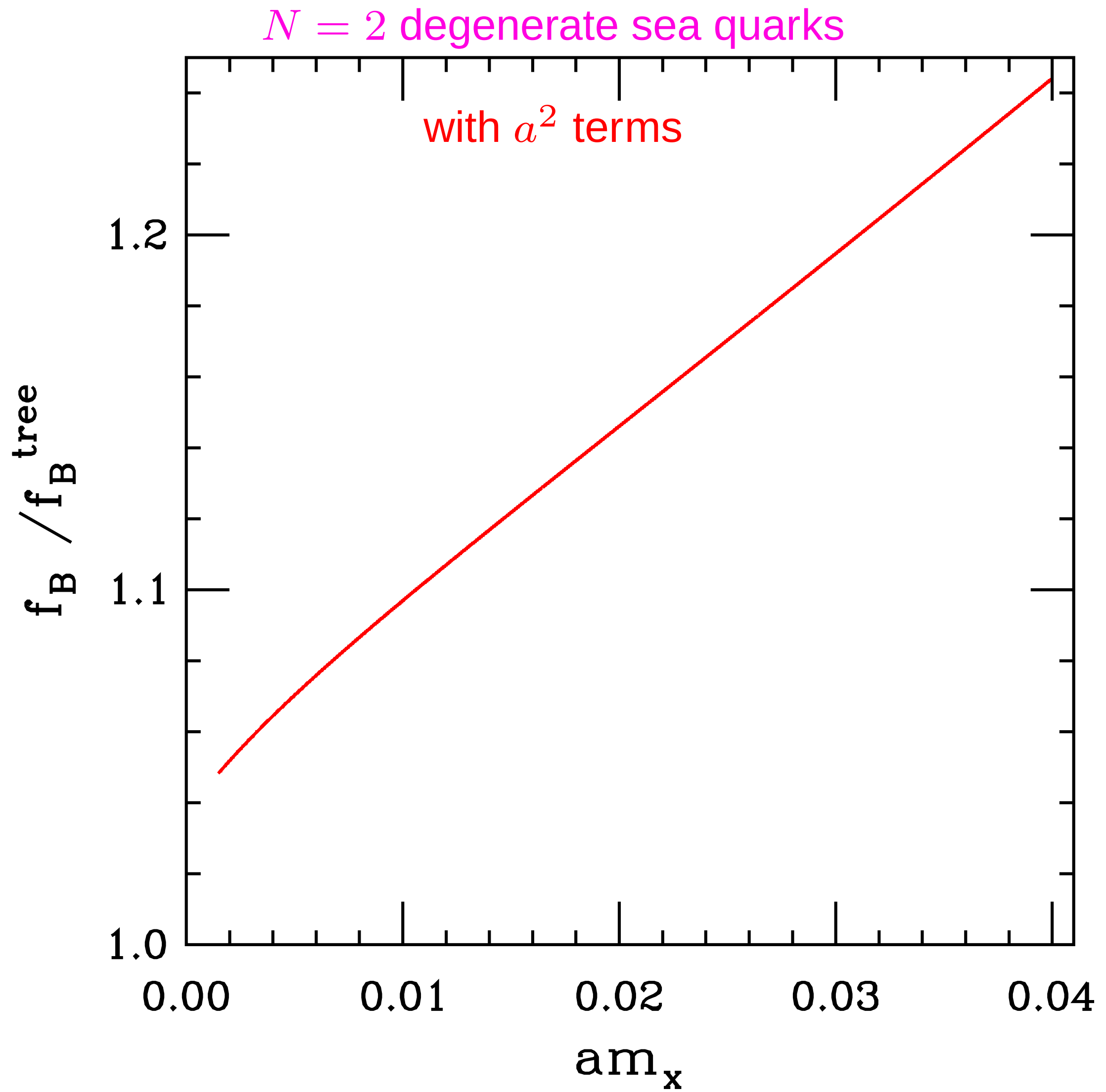
$$m_u = m_d \equiv m_l$$

$$\delta f_B = \left(\frac{1+3g_\pi^2}{2}\right)\Big\{-\frac{1}{16}\sum_t \left[2\ell(m_{\pi_t}^2)+\ell(m_{K_t}^2)\right] - \frac{1}{2}\ell(m_{\pi_I^0}^2) + \frac{1}{6}\ell(m_{\eta_I}^2)$$

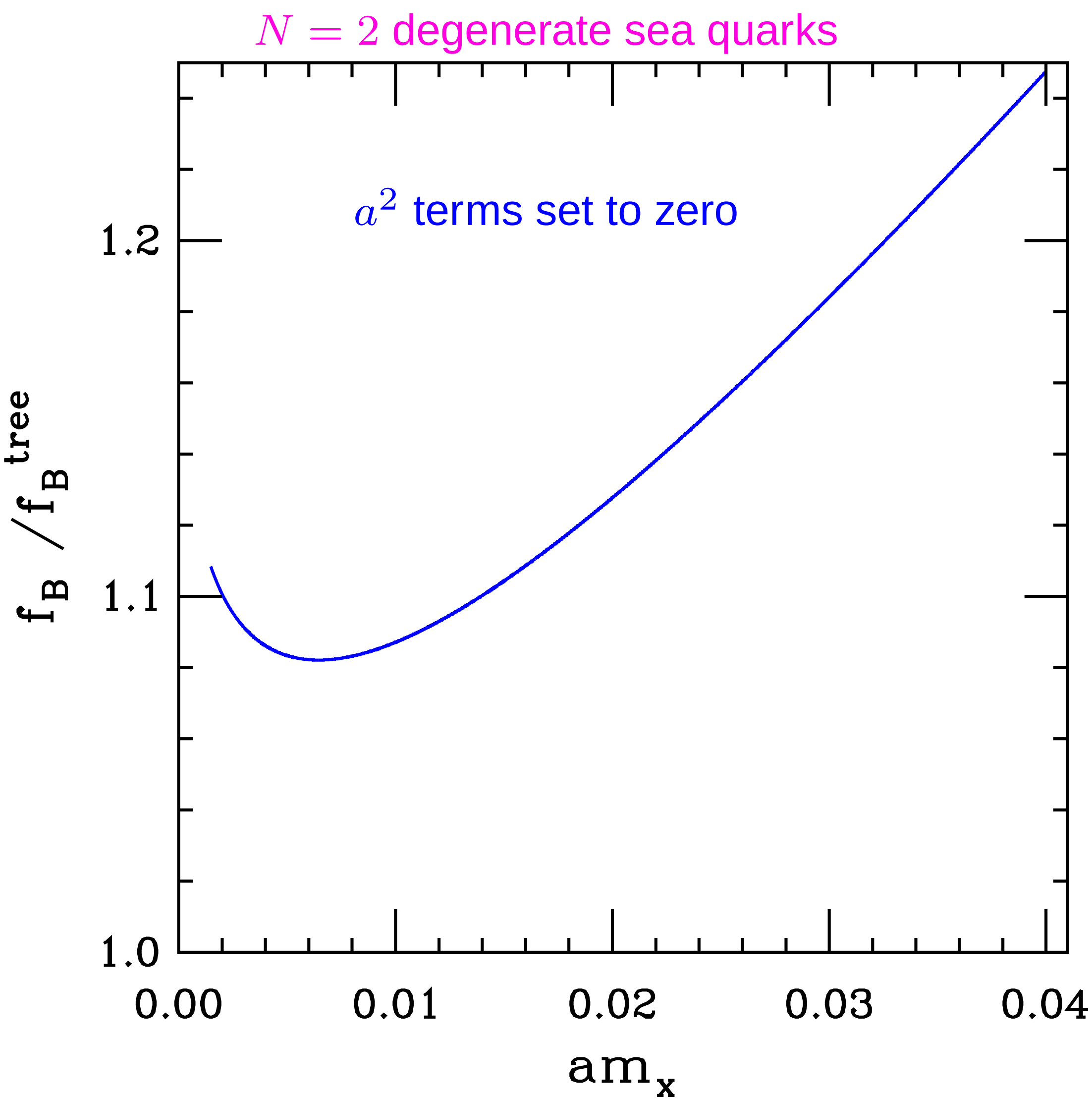
$$-a^2\delta_V'\left[\frac{(m_{\pi_V^0}^2 - m_{S_V}^2)}{(m_{\pi_V^0}^2 - m_{\eta_V}^2)(m_{\pi_V^0}^2 - m_{\eta_V'}^2)}\ell(m_{\pi_V^0}^2) + (\pi_V^0 \to \eta_V \to \eta_V')\right]$$

$$+[V \to A] + c_1(2m_l + m_s) + c_2 m_l + c_a a^2\Big\}$$

f_B with $am = 0.010$

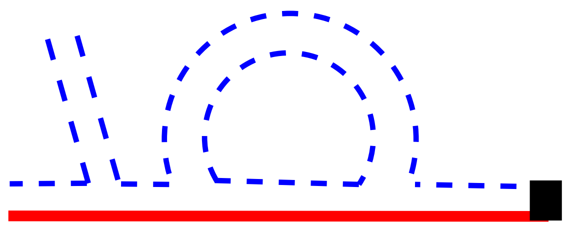


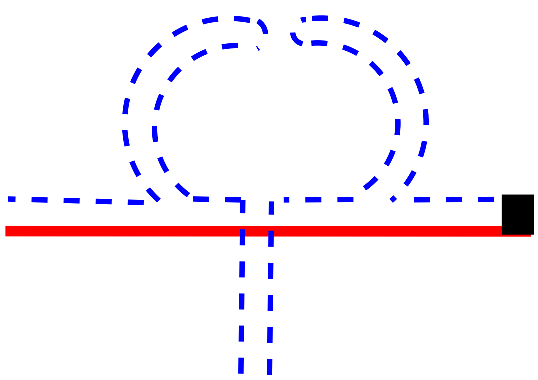
f_B **with** $am = 0.010$



Form Factors

- Continuum PQχPT results with $N_{\rm sea}$ degenerate sea quarks can be easily generalized to SχPT:
  1. Terms $\propto N_{\rm sea} \longrightarrow$ average over tastes.
  2. Terms $\propto \dfrac{1}{N_{\rm sea}}$ are disconnected; in SχPT we have S, V, and A pieces.
  3. One must be careful of possible sign changes when commuting taste matrices.
- With this in mind, it is straightforward to write down the SχPT expressions for the form factors for $B(D) \to \pi(K)\ell\nu$ decays.





Taste-breaking Four-quark operators

- There are three types of four-quark operators that can possibly contribute to $\mathcal{V}_H$:

1. $\overline{Q}(\gamma_S \otimes I)Q\overline{Q}(\gamma_{S'} \otimes I)Q$
   Taste singlet: No taste violations from these operators

2. $\overline{Q}(\gamma_S \otimes I)Q\overline{q}_j(\gamma_{S'} \otimes \xi_I)q_j$
   Correct the masses of the heavy-lights at $\mathcal{O}(a^2)$, do not involve the pions at this order.

3. $\overline{q}_i(\gamma_S \otimes \xi_T)q_i\overline{q}_j(\gamma_{S'} \otimes \xi_{T'})q_j$
   Give rise to the terms in $\mathcal{V}_H$.

Potential with Heavy-lights

- The terms in $\mathcal{V}_H$ have one of two forms:

$$\overline{H}_a H_b O_{ba}$$
$$\overline{H}_a H_a O_{bb}$$

- There are 8 operators O_{ba} which contribute, giving 16 new terms.
- For example:

$$\begin{aligned} O^1_{ba} &= (\sigma\xi^{(3)}_5 \Sigma^\dagger \xi^{(3)}_5 \sigma)_{ba} \\ O^7_{ba} &= (\sigma\xi^{(3)}_\nu \sigma)_{ba}\, \mathrm{Tr}(\xi^{(3)\nu}\Sigma^\dagger) \end{aligned}$$

$$\xi^{(3)}_\mu = \begin{pmatrix} \xi_\mu & 0 & 0 \\ 0 & \xi_\mu & 0 \\ 0 & 0 & \xi_\mu \end{pmatrix}$$

Conclusions

- We can now include heavy-light mesons in SχPT calculations.
- $\mathcal{V}_H$ is not necessary for many important quantities at one-loop order.
- Calculations for f_B and form factors have been completed.
- Other calculations, such as B parameters or other relevant quantities are now straightfoward.